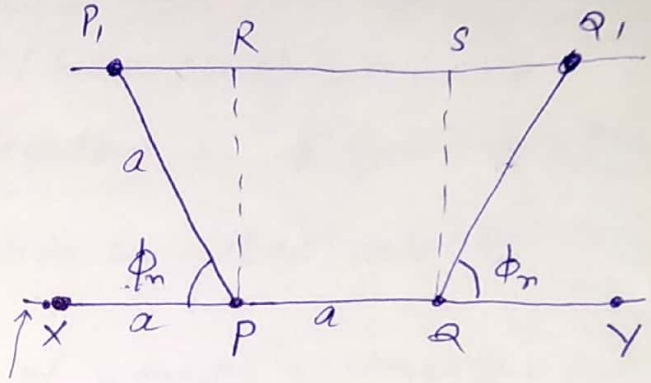
Why five fold rotation axis's not possible ?

The axis of symmetry may be defined as a line (passing through a lattice point) such that crystal assumes a congruent position for every rotation of $\left(\frac{2\pi}{n}\right)$.

$n \rightarrow$ fold of the axis.

Let axis has n fold rotation axis's of symmetry.

Rotate vectors PX & QY by $\phi_n = \frac{2\pi}{n}$ in Clockwise & anticlockwise direction respectively



Lattice row XPQY

Lattice translation = a

If lattice posses n-fold rotation axis's of symmetry P1 & Q1, they should be lattice points.

Rotation operation must leave the lattice Invariant

$\rightarrow$ P_1Q_1 must be $\parallel^l$ to PQ and $P_1Q_1 = ma$ ↑ integer

$$\cos\phi_n = \frac{P_1R}{PP_1} \Rightarrow P_1R = PP_1\cos\phi_n = a\cos\phi_n$$

and $SQ_1 = a\cos\phi_n$

$$P_1Q_1 = P_1R + RS + SQ_1 = a\cos\phi_n + a + a\cos\phi_n$$

$$ma = 2a\cos\phi_n + a \Rightarrow 1 + \cos\phi_n = m$$

$$\boxed{\cos\phi_n = \frac{m-1}{2} = \frac{N}{2}}$$

N = integer

$\boxed{-1 \leq N/2 \leq +1}$

N	$\frac{N}{2}$	$\cos\phi_n$	ϕ_n	$\frac{360^\circ}{n}$
-2	-1	-1	180°	2
-1	-1/2	-1/2	120°	3
0	0	0	90°	4
+1	+1/2	+1/2	60°	6
+2	+1	+1	360°	1

$\Rightarrow n \neq 5$

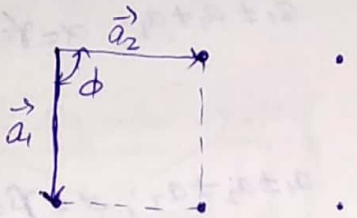
Two-Dimensional lattice Types!

5 distinct lattice type in 2-D (Bravais lattice)

Oblique lattice + 4 special lattice (Page 2)

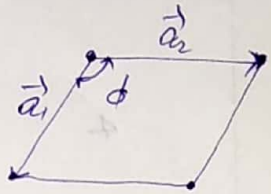
Special lattice of oblique type invariant under rotation of $\frac{2\pi}{3}$, $\frac{2\pi}{4}$ or $\frac{2}{6}$

or under mirror reflection.



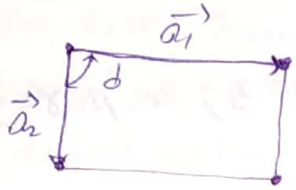
(a) Square lattice

$|\vec{a}_1| = |\vec{a}_2|, \phi = 90^\circ$



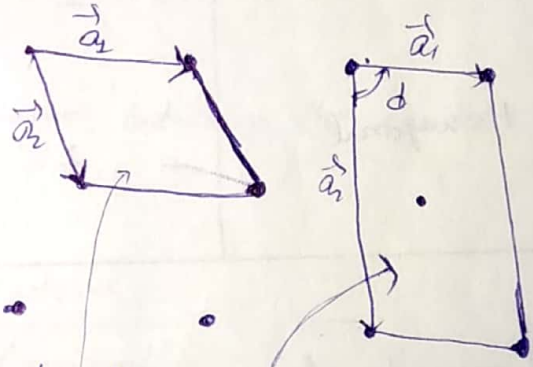
(b) Hexagonal lattice

$|\vec{a}_1| = |\vec{a}_2|, \phi = 120^\circ$



(c) Rectangular lattice

$|\vec{a}_1| \neq |\vec{a}_2|, \phi = 90^\circ$



(d) Centered rectangular lattice:

$|\vec{a}_1| \neq |\vec{a}_2|; \phi = 90^\circ$

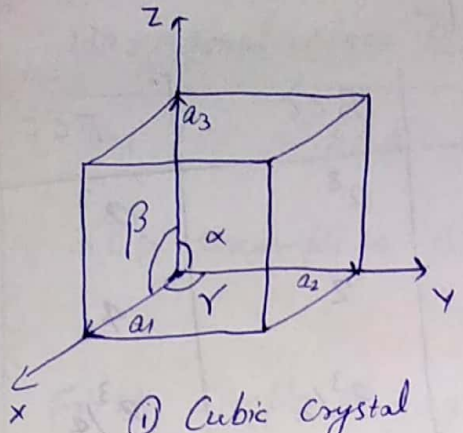
Primitive cell

Three Dimensional lattice type :-

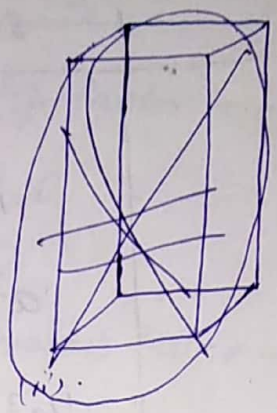
7 Crystal System ⟶ 14 Bravais lattice

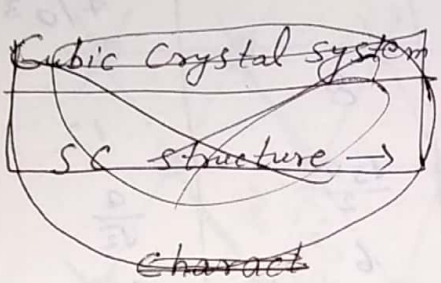
System	Number of Lattices	Restrictions on conventional cell axes and angles
Triclinic	1	$a_1 \neq a_2 \neq a_3$; $\alpha \neq \beta \neq \gamma$
Monoclinic	2	$a_1 \neq a_2 \neq a_3$; $\alpha = \gamma = 90^\circ \neq \beta$
Orthorhombic	4	$a_1 \neq a_2 \neq a_3$; $\alpha = \beta = \gamma = 90^\circ$
Tetragonal	2	$a_1 = a_2 \neq a_3$; $\alpha = \beta = \gamma = 90^\circ$
Cubic	3	$a_1 = a_2 = a_3$; $\alpha = \beta = \gamma = 90^\circ$
Trigonal	1	$a_1 = a_2 = a_3$; $\alpha = \beta = \gamma = 90^\circ$
Hexagonal	1	$a_1 = a_2 \neq a_3$; $\alpha = \beta = 90^\circ$, $\gamma = 120^\circ$

Conventional unit cell: The conventional unit cell is generally chosen to be bigger than the primitive cell and to have required symmetry.



① Cubic Crystal System





Some Important Crystal structure terms

(i) Coordination number: Coordination number is defined as number of equidistant nearest neighbours that an atom has in the given structure. Greatest the Coordination number, the more closely packed up will be the structure.

(ii) Nearest neighbour distance: the distance between the centres of two nearest neighbouring atoms. (2r)

(iii) Packing factor = $\frac{\text{volume of atoms}}{\text{volume of unit cell}}$